## Asymmetric Di-jet Production in Polarized Hadronic Collisions

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## Abstract

Using the collinear QCD factorization approach, we study the single-transverse-spin dependent cross section  $\Delta\sigma(S_{\perp})$  for the hadronic production of two jets of momenta  $P_1 = P + q/2$  and  $P_2 = -P + q/2$ . We consider the kinematic region where the transverse components of the momentum vectors satisfy  $P_{\perp} \gg q_{\perp} \gg \Lambda_{\rm QCD}$ . For the case of initial-state gluon radiation, we show that at the leading power in  $q_{\perp}/P_{\perp}$  and at the lowest non-trivial perturbative order, the dependence of  $\Delta\sigma(S_{\perp})$  on  $q_{\perp}$  decouples from that on  $P_{\perp}$ , so that the cross section can be factorized into a hard part that is a function only of the single scale  $P_{\perp}$ , and into perturbatively generated transverse-momentum dependent (TMD) parton distributions with transverse momenta  $k_{\perp} = \mathcal{O}(q_{\perp})$ .

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1. Introduction. Single-transverse-spin asymmetries (SSAs) in high-energy hadronic reactions with one transversely polarized hadron were first observed more than three decades ago [1]. The SSA is defined as  $A_N \equiv (\sigma(S_\perp) - \sigma(-S_\perp))/(\sigma(S_\perp) + \sigma(-S_\perp))$ , the ratio of the difference and the sum of (differential) cross sections when the hadron's spin vector,  $S_\perp$ , is flipped. Recent experimental measurements of SSAs both in polarized hadronic collisions [2, 3] and in semi-inclusive lepton-nucleon deep inelastic scattering (SIDIS) [4] have renewed the interest in investigating the origin of SSAs in Quantum Chromodynamics (QCD) [5].

It is believed that some SSAs are a consequence of the partons' transverse motion inside the polarized hadron. The momentum scale of this transverse motion is a typical hadronic scale,  $\langle k_{\perp} \rangle \sim 1/\text{fm} \sim \Lambda_{\text{QCD}}$ . For observables with only one hard scale  $Q \gg \Lambda_{\text{QCD}}$ , the SSA should be proportional to  $\langle k_{\perp} \rangle / Q$  [6, 7]. Such observables only probe an averaged effect of the partons' transverse motion. However, for observables characterized by more than one physical scale, SSAs may directly probe the partons' transverse motion. For example, in the case of Drell-Yan hadronic production of a lepton pair of large invariant mass Q and transverse momentum  $q_{\perp} \ll Q$ , the pair probes the (anti-) quark's transverse motion at the scale  $q_{\perp}$ , while the invariant mass Q of the pair sets the hard scale of the collision [8]. In this letter, we study the SSA in hadronic production of two jets:  $A(P_A, S_{\perp}) + B(P_B) \rightarrow J_1(P_1) + J_2(P_2) + X$ , with the jet momenta  $P_1 \equiv P + q/2$  and  $P_2 \equiv -P + q/2$  [9, 10, 11]. Unlike in the Drell-Yan process or in SIDIS, the SSA in di-jet production can be generated by both initial- and final-state interactions. We emphasize that measurements of the SSA for di-jet production have begun at RHIC [12], complementing the measurements in SIDIS.

We are interested here in deriving a QCD formalism for the single transverse-spin dependent cross section,  $\Delta\sigma(S_{\perp}) = (\sigma(S_{\perp}) - \sigma(-S_{\perp}))/2$ , that is valid in the kinematic region  $P_{\perp} \gg q_{\perp} \gtrsim \Lambda_{\rm QCD}$ , where  $P_{\perp}$  and  $q_{\perp}$  are the transverse components of the momenta P and q, respectively, so that the SSA provides direct information on the partons' transverse motion. We first consider the region  $P_{\perp} \gg q_{\perp} \gg \Lambda_{\rm QCD}$ , where both observed momentum scales are much larger than the typical hadronic scale  $\Lambda_{\rm QCD}$ . We calculate  $\Delta\sigma(S_{\perp})$  in terms of the collinear QCD factorization approach, which is expected to be valid in this region [13]. In this approach, the incoming partons are approximated to be collinear to the corresponding initial hadrons, and the leading-order partonic processes produce two back-to-back jets with zero momentum imbalance. The di-jet momentum imbalance,  $\vec{q}_{\perp} = \vec{P}_{1\perp} + \vec{P}_{2\perp}$ , has to be perturbatively generated by radiating an additional hard parton.

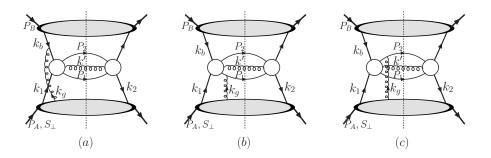


FIG. 1: Sample diagrams for quark-quark scattering contributing to  $\Delta\sigma(S_{\perp})$  through an initial-state interaction (a), and through final-state interactions with jet  $P_1$  (b) and jet  $P_2$  (c).

In this letter, we concentrate on the physics issues related to di-jet production, and restrict ourselves to the case that the leading contribution in the expansion of the partonic scattering in  $q_{\perp}/P_{\perp}$  involves a hard  $qq' \to qq'$  subprocess. We consider the jet imbalance  $q_{\perp}$  to be generated by gluon radiation off the initial quarks. We derive the corresponding leading-order contribution to  $\Delta\sigma(S_{\perp})$ , and demonstrate that the perturbatively calculated partonic parts can be further factorized into a single-scale  $(P_{\perp})$  hard part and perturbatively generated transverse-momentum dependent (TMD) parton distributions with transverse momenta  $k_{\perp} = \mathcal{O}(q_{\perp})$ . We find that the complete contributions from all other partonic subprocesses at the leading order have the same factorization property. These will be discussed in a forthcoming publication [14]. The factorization of the physics at scale  $P_{\perp}$  from that at scale  $q_{\perp}$  that we find when  $q_{\perp} \ll P_{\perp}$ , is consistent with a more general TMD factorization formula for the SSA in the di-jet momentum imbalance.

2. Single transverse-spin dependent cross section. When both  $P_{\perp}$  and  $q_{\perp}$  are much larger than  $\Lambda_{\rm QCD}$ , a nonvanishing single transverse-spin dependent cross section  $\Delta\sigma(S_{\perp})$  is generated by the Efremov-Teryaev-Qiu-Sterman (ETQS) mechanism [6, 7] in the collinear factorization approach. The calculation of  $\Delta\sigma(S_{\perp})$  then requires to evaluate partonic processes with 3-parton initial- and final-states [13]. In Fig. 1 we show generic diagrams for the quark-quark scattering channel that contribute to  $\Delta\sigma(S_{\perp})$  through initial- and final-state interactions with the gluon of momentum  $k_g$ , which is needed for generating the phase required for a nonvanishing SSA [6, 7]. Radiation of a hard gluon of momentum k' into the final state generates the jet imbalance  $q_{\perp}$ . The blob in the center represents tree-level Feynman diagrams with the given initial- and final-state partons. In the ETQS formalism, the contribution of the subprocess  $(g)qq' \to qq'g$  to  $\Delta\sigma(S_{\perp})$ , shown in Fig. 1, is generically

given by

$$\frac{d\Delta\sigma(S_{\perp})_{(qq')}}{dy_1 dy_2 dP_{\perp}^2 d^2 \vec{q}_{\perp}} = \int \frac{dx'}{x'} dx_1 dx_2 T_F(x_1, x_2) q'(x') 
\times \frac{1}{16s(2\pi)^4} \delta((k')^2) \mathcal{H}_{(g)qq' \to qq'g},$$
(1)

where  $y_1$  and  $y_2$  are the rapidities of the two jets,  $s = (P_A + P_B)^2$ , and  $\mathcal{H}$  represents a partonic hard part. q'(x') is the usual quark distribution at momentum fraction x' in the incoming hadron B.  $x_1$  and  $x_2$  are the momentum fractions of the quarks from the polarized hadron A on the two sides of the cut shown in Fig. 1, and  $T_F(x_1, x_2)$  is the corresponding twist-three quark-gluon correlation function, extracted from the lower blob in the figure [7, 15]:

$$T_{F}(x_{1}, x_{2}) \equiv \int \frac{d\zeta^{-}d\eta^{-}}{4\pi} e^{i(x_{1}P_{A}^{+}\eta^{-} + (x_{2} - x_{1})P_{A}^{+}\zeta^{-})}$$

$$\times \epsilon_{\perp}^{\beta\alpha} S_{\perp\beta} \langle P_{A}, S | \overline{\psi}(0) \mathcal{L}(0, \zeta^{-}) \gamma^{+}$$

$$\times gF_{\alpha}^{+}(\zeta^{-}) \mathcal{L}(\zeta^{-}, \eta^{-}) \psi(\eta^{-}) | P_{A}, S \rangle , \qquad (2)$$

where  $\mathcal{L}$  is the proper gauge link to make the matrix element gauge invariant, and where the sums over color and spin indices are implicit. In Eq. (1) and the rest of this paper, the dependence on factorization and renormalization scales is suppressed.

Equation (1) applies when  $q_{\perp} \sim P_{\perp}$ . Our goal is now to investigate the leading structure that emerges from Eq. (1) when  $q_{\perp} \ll P_{\perp}$ . In this limit the gluon of momentum k' is radiated either nearly collinearly from one of the external quark legs and/or is soft. In the present work, we only discuss collinear emission by one of the *initial* quarks. This radiation is the most interesting from the point of view of studying the factorization properties of the cross section at small  $q_{\perp}$ , because TMD factorization can only hold if the initial-state collinear radiation leads to a certain specific structure, as we shall discuss below. Since we are considering the production of jets (as opposed to that of two specific hadrons), collinear radiation from *final*-state quarks becomes part of the jet and will not produce leading behavior in  $q_{\perp}/P_{\perp}$ . On the other hand, large-angle soft gluons produced by the interference of initial- and final state radiation may give leading contributions, through a so-called soft factor. We leave the detailed study of the soft factor to future work, but will briefly return to it later.

As we mentioned above, the strong interaction phase necessary for a nonvanishing  $\Delta \sigma(S_{\perp})$  arises from the interference between the imaginary part of the partonic scattering amplitude

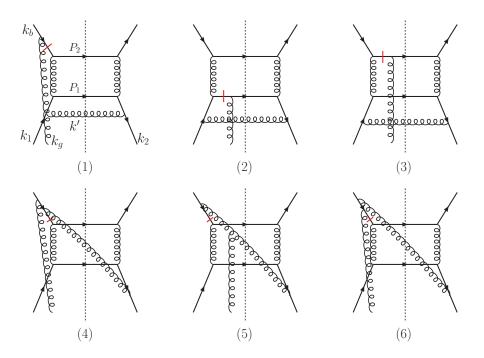


FIG. 2: Sample diagrams for the soft-pole (1-3) and hard-pole (4-6) contributions. We have indicated the momentum k' of the radiated gluon that produces the jet imbalance  $q_{\perp}$ , and the momentum  $k_g$  of the additional gluon from the polarized proton. The pole of each diagram is taken from the propagator with a short bar. Diagram (1) is an example of an initial-state interaction, and (2) and (3) show final-state interactions. Diagrams (4-6) show the complete set of diagrams for one hard pole, indicated again by the short bar.

with the extra polarized gluon of momentum  $k_g = x_g P_A$  and the real scattering amplitude without the gluon in Fig. 1. The imaginary part comes from taking the pole of the parton propagator associated with the integration over the gluon momentum fraction  $x_g = x_2 - x_1$ . For a process with two physical scales,  $P_{\perp}$  and  $q_{\perp}$ , tree scattering diagrams in Fig. 1 have two types of poles, corresponding to  $x_g = 0$  ("soft-pole") [7] and  $x_g \neq 0$  ("hard-pole") [8]. With the extra initial-state gluon attachment of momentum k', there are many more diagrams that contribute to  $\Delta \sigma(S_{\perp})$  in comparison to the spin-averaged cross section [14]. In Fig. 2, we show some sample diagrams that give the leading soft-pole (1-3) and hard pole (4-6) contributions to  $\Delta \sigma(S_{\perp})$  at  $q_{\perp} \ll P_{\perp}$ . By using the power counting technique [16], we are able to classify all Feynman diagrams into different groups [14]. For example, diagrams (1-3) only make leading contributions when the momentum k' of the radiated gluon is parallel to  $P_A$ , whereas diagrams (4-6) can give a leading contribution when k' is either parallel to  $P_A$  or

to  $P_B$ . Beyond that, the calculation of the soft-pole and hard-pole contributions to  $\Delta\sigma(S_\perp)$  follows the same procedure as introduced in Ref. [8] for the SSA in Drell-Yan and SIDIS, because the kinematic limit considered is similar. In order to extract the contributions to  $\Delta\sigma(S_\perp)$  in Eq. (1), we need to convert the extra gluon field operator in the hadronic matrix element of the polarized hadron in Fig. 1 to a field strength operator in the definition of  $T_F(x_1, x_2)$  in Eq. (2). Working in Feynman gauge, we first give the initial-state collinear partons from the polarized hadron a small transverse momentum,  $k_i = x_i P_A + k_{i\perp}$  with i = 1, 2, and then expand the calculated partonic scattering amplitudes around  $k_{i\perp} = 0$ , or equivalently,  $k_{g\perp} = k_{2\perp} - k_{1\perp} = 0$ . The contribution to  $\Delta\sigma(S_\perp)$  arises from terms linear in  $k_{g\perp}$ . After summing up all contributions [14], we obtain the total leading-power contribution to  $\Delta\sigma(S_\perp)$  from the  $(g)qq' \to qq'g$  partonic subprocess in the  $q_\perp/P_\perp$  expansion:

$$\frac{d\Delta\sigma(S_{\perp})_{(qq')}}{dy_1 dy_2 dP_{\perp}^2 d^2 \vec{q}_{\perp}} \bigg|_{P_{\perp} \gg q_{\perp} \gg \Lambda_{\text{QCD}}} = -\frac{\epsilon^{\alpha\beta} S_{\perp}^{\alpha} q_{\perp}^{\beta}}{(\vec{q}_{\perp}^2)^2} H_{qq' \to qq'}^{\text{Sivers}} \times \frac{\alpha_s}{2\pi^2} x_a x_b \left[ q'(x_b) \int \frac{dx}{x} \mathcal{A} + T_F(x_a, x_a) \int \frac{dx'}{x'} \mathcal{B} \right], \tag{3}$$

where

$$\mathcal{A} = \left\{ x \frac{\partial}{\partial x} T_F(x, x) \frac{1}{2N_c} \left[ 1 + \xi^2 \right] + T_F(x, x) \frac{1}{2N_c} \left[ \frac{2\xi^3 - 3\xi^2 - 1}{1 - \xi} \right] + T_F(x, \xi x) \left( \frac{1}{2N_c} + C_F \right) \left[ \frac{1 + \xi}{1 - \xi} \right] \right\} ,$$

$$\mathcal{B} = q'(x') C_F \left[ \frac{1 + \xi'^2}{1 - \xi'} \right] ,$$
(5)

where  $\xi = x_a/x$  and  $\xi' = x_b/x'$  with  $x_a = \frac{P_\perp}{\sqrt{s}} (e^{y_1} + e^{y_2})$  and  $x_b = \frac{P_\perp}{\sqrt{s}} (e^{-y_1} + e^{-y_2})$ . The above relations are regularized at the integration limits by "plus"-distributions [8]. The single-scale partonic hard part is given by

$$H_{qq' \to qq'}^{\text{Sivers}}(\hat{s}, \hat{t}, \hat{u}) = \frac{\alpha_s^2 \pi}{\hat{s}^2} \left[ \frac{N_c^2 - 5}{4N_c^2} \right] \frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2} , \tag{6}$$

where the use of the superscript "Sivers" will become clear in the next section, and where the partonic Mandelstam variables are given as  $\hat{s} = x_a x_b s$ ,  $\hat{t} = -P_{\perp}^2 (e^{y_2 - y_1} + 1)$ , and  $\hat{u} = -P_{\perp}^2 (e^{y_1 - y_2} + 1)$ . We note that to the leading power in  $q_{\perp}/P_{\perp}$ , we have  $P_1 = P_{\perp} (e^{y_1}/\sqrt{2}, e^{-y_1}/\sqrt{2}, 1)$  and  $P_2 = P_{\perp} (e^{y_2}/\sqrt{2}, e^{-y_2}/\sqrt{2}, -1)$  for the jet momenta in light-cone coordinates.

The hard part for the partonic channel we have considered,  $H_{qq'\to qq'}^{\text{Sivers}}$ , is very similar to the spin-averaged partonic differential cross section  $d\hat{\sigma}/d\hat{t}$  [14]. The only difference is the color factor in square brackets. In fact, the color factor for  $H_{qq'\to qq'}^{\text{Sivers}}$  is equal to a sum of three color factors:  $C_I + C_{F_1} + C_{F_2}$ , corresponding to color factors of scattering amplitudes when the initial-state gluon of momentum  $k_g$  is attached to the initial-state incoming quark of momentum  $k_b$ , the final-state quark with  $P_1$ , or the final-state quark with  $P_2$ , respectively. For the  $(g)qq'\to qq'g$  subprocess, we have [14]  $C_I = -\frac{1}{2N_C^2}$ ,  $C_{F_1} = -\frac{1}{4N_C^2}$ , and  $C_{F_2} = \frac{N_c^2-2}{4N_C^2}$ . As a result, the final-state interaction with the jet of momentum  $P_2$  dominates, and the overall color factor  $C_I + C_{F_1} + C_{F_2}$  has a sign opposite to that of  $C_I$  alone.

3. Factorization in terms of TMD distributions. When  $q_{\perp} \ll P_{\perp}$ , the di-jet production and Drell-Yan process at low transverse momentum  $(q_T \ll Q)$  considered in Ref. [8] share very similar kinematics. The difference is that the Drell-Yan process has only initial-state interactions while for di-jet production both initial- and final-state interactions are present. It is known [17, 18] that when  $q_{\perp} \ll Q$ , the Drell-Yan cross section in leading order in  $q_{\perp}/Q$  can be calculated from a generalized QCD factorization formula involving the TMD parton distributions. It is natural to ask if the generalized QCD factorization formula can be extended to the di-jet cross section for  $q_{\perp} \ll P_{\perp}$ .

When  $q_{\perp} \gg \Lambda_{\rm QCD}$ , TMD parton distributions can be calculated in perturbative QCD (pQCD) from hard radiation and parton splitting [8]. The unpolarized TMD quark distribution is well known and given by [8]

$$q(x_b, q_\perp) = \frac{\alpha_s}{2\pi^2} \frac{1}{\vec{q}_\perp^2} \int \frac{dx'}{x'} \left[ \mathcal{B} + \cdots \right] , \qquad (7)$$

where  $\mathcal{B}$  is given in Eq. (5), and where the ellipses denote terms proportional to  $\delta(1-\xi)$  and terms generated by gluon splitting. The transverse-spin dependent TMD quark distribution,  $q_T(x_a, q_\perp)$ , known as the Sivers function [19], can also be calculated in pQCD and is given in terms of the twist-3 quark-gluon correlation function  $T_F$  as [8]:

$$q_T^{\text{SIDIS}}(x_a, q_\perp) = -\frac{\alpha_s}{4\pi^2} \frac{2M_P}{(\vec{q}_\perp^2)^2} \int \frac{dx}{x} \left[ \mathcal{A} + \dots \right] , \qquad (8)$$

where  $\mathcal{A}$  is given in Eq. (4),  $M_P$  is a hadron mass scale introduced to keep  $q(x_b, q_\perp)$  and  $q_T(x_a, q_\perp)$  at the same dimension, and where the ellipses as above denote terms proportional to  $\delta(1-\xi)$  and terms not relevant to the following discussion. In Eq. (8), the superscript "SIDIS" indicates the Sivers function for the SIDIS process, which has an opposite sign from

that for the Drell-Yan process given in Ref. [8], due to the difference in the directions of the gauge link that defines the TMD quark distributions [20, 21, 22, 23].

One of the important features of the di-jet cross section calculated above to leading order in  $q_{\perp}/P_{\perp}$  is the separation of two observed physical scales,  $P_{\perp}$  and  $q_{\perp}$ . We can use Eqs. (7) and (8) to rewrite the single transverse-spin dependent di-jet cross section in Eq. (3) in terms of a single-scale hard factor,  $H^{\text{Sivers}}$ , which is a function of  $P_{\perp}$ , and of the  $q_{\perp}$ -dependent perturbatively generated TMD parton distributions:

$$\frac{d\Delta\sigma(S_{\perp})_{(qq')}}{dy_{1}dy_{2}dP_{\perp}^{2}d^{2}\vec{q}_{\perp}}\Big|_{P_{\perp}\gg q_{\perp}\gg\Lambda_{\rm QCD}} = \epsilon^{\alpha\beta}S_{\perp}^{\alpha}q_{\perp}^{\beta}H_{qq'\to qq'}^{\rm Sivers}$$

$$\times \left[\frac{1}{M_{P}}\left(x_{b}q'(x_{b})\right)\left(x_{a}q_{T}^{\rm SIDIS}(x_{a},q_{\perp})\right)\right]$$

$$-\frac{1}{\vec{q}_{\perp}^{2}}\left(x_{b}q'(x_{b},q_{\perp})\right)\left(x_{a}T_{F}(x_{a},x_{a})\right)\right].$$
(9)

Using the leading order relation [23]

$$\frac{1}{M_P} \int d^2k_{\perp} \, \vec{k}_{\perp}^2 \, q_T^{\text{SIDIS}}(x, k_{\perp}) = -T_F(x, x) \,, \tag{10}$$

we find that our result in Eq. (9), when the contributions from all other partonic subprocesses at the same order [14] are added, is consistent with the leading-order term of a more general factorization formula in terms of TMD parton distributions:

$$\frac{d\Delta\sigma(S_{\perp})}{dy_{1}dy_{2}dP_{\perp}^{2}d^{2}\vec{q}_{\perp}} = \frac{\epsilon^{\alpha\beta}S_{\perp}^{\alpha}q_{\perp}^{\beta}}{\vec{q}_{\perp}^{2}} \sum_{ab} \int d^{2}k_{1\perp}d^{2}k_{2\perp}d^{2}\lambda_{\perp}$$

$$\times \frac{\vec{k}_{1\perp} \cdot \vec{q}_{\perp}}{M_{P}} x_{a} q_{Ta}^{\text{SIDIS}}(x_{a}, k_{1\perp}) x_{b} f_{b}^{\text{SIDIS}}(x_{b}, k_{2\perp})$$

$$\times \left[ S_{ab \to cd}(\lambda_{\perp}) H_{ab \to cd}^{\text{Sivers}}(P_{\perp}^{2}) \right]_{c} \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\lambda}_{\perp} - \vec{q}_{\perp}) , \tag{11}$$

where apart from the functions already given,  $f_b^{\text{SIDIS}}$  denotes the unpolarized TMD quark distribution and  $S_{ab\to cd}$  is the soft factor mentioned above [8, 14]. Because of the color flow into the jets, the product of the soft and hard factors will involve a sum over separate color amplitudes in the full factorization formalism [24, 25], which has been represented by a trace [ $]_c$  in color space in the above equation. Our calculation of initial-state collinear gluon radiation described above would not be sensitive to this complexity of the color flow, but we emphasize that the definition of the parton distributions cannot be affected by it [14, 25].

In Eq. (11), we have chosen TMD parton distributions defined in SIDIS because of the dominance of final-state interactions. Choosing TMD parton distributions defined according to the Drell-Yan process would change the sign of the partonic hard factors, but not affect the overall sign of the physical cross section. Based on our explicit calculation here and the generalized factorization property of the Drell-Yan process at low  $q_{\perp}$  [17, 18], and because of the similarity in kinematics between two processes, we expect the generalized factorization formula in Eq. (11) to be valid for describing the single-transverse-spin dependent cross section for the di-jet momentum imbalance in hadronic collisions in the kinematic region where  $P_{\perp} \gg q_{\perp} \gtrsim \Lambda_{\rm QCD}$ .

A key feature of the factorization is that the perturbatively calculated short-distance hard factors should not be sensitive to details of the factorized long distance physics. We tried, as a test, to derive all short-distance hard factors by using this factorization formula and the Brodsky-Hwang-Schmidt model for SSAs [20]. We were able to recover all hard-scattering factors in this way. We note that the same hard factors  $H_{ab\to cd}^{\rm Sivers}$  as above were also found for the weighted (integrated) SSA for di-jet production in [10]. A detailed comparison between the approach of [10] and ours will be presented in Ref. [14]. An all-order proof of the above factorization formula, if correct, is still needed and is beyond the scope of this paper.

4. Summary. We have studied the single-transverse-spin dependent cross section  $\Delta\sigma(S_{\perp})$  for di-jet production momentum imbalance in high-energy hadronic collisions in a kinematic region where  $P_{\perp}\gg q_{\perp}\gg \Lambda_{\rm QCD}$ , and calculated contributions from both initial-and final-state interactions. At the leading order in  $q_{\perp}/P_{\perp}$ , the  $q_{\perp}$  and  $P_{\perp}$  dependences in our calculated results are decoupled and can be factorized into a single-scale hard factor that depends on  $P_{\perp}$ , and into perturbatively generated TMD parton distributions. This factorization occurs for each partonic channel. Overall, final-state interactions turn out to give the dominant contribution to  $\Delta\sigma(S_{\perp})$  [14]. We therefore expect the SSA in di-jet production to have the same sign as the Sivers asymmetry in SIDIS.

We have found that our results are consistent with a more general TMD factorization formula, given in Eq. (11), which we propose to be the correct approach to describing the single-transverse spin asymmetry in di-jet production at hadron colliders when  $P_{\perp} \gg q_{\perp}$ . We emphasize that obviously the result of our first-order calculation is not able to actually prove this factorization, but should rather be regarded as a "necessary condition" for such a factorization to hold. A full proof remains an important challenge for future work. Also,

we recall that we have limited ourselves to the case of collinear initial-state gluon radiation. The effects of large-angle soft-gluon emission have been neglected, even though we have indicated their likely role in Eq. (11). A proof of TMD factorization in this process would naturally incorporate a study of this soft factor.

If the proposed factorization formula is valid, the di-jet momentum imbalance at RHIC could be described by the same TMD parton distributions as those used to describe the SIDIS and Drell-Yan processes. Because both initial- and final-state interactions are present, the di-jet momentum imbalance is sensitive to different short distance dynamics, and it will be an excellent process to test QCD factorization and the universality of the TMD parton distributions. In addition it should give valuable information on the partons' transverse motion in the nucleon.

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